Mathematical Biology Take Home Final Exam
Due December 8, 2005 at 9am.

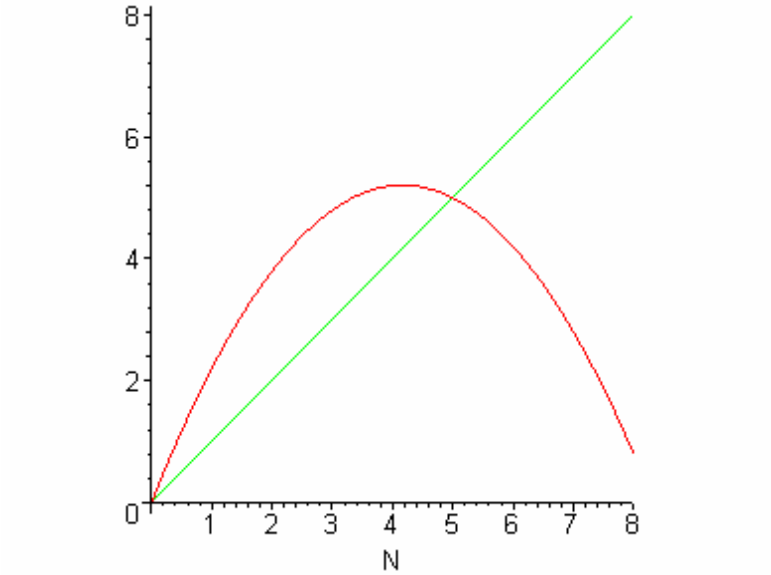
You may use Maple, class notes, the text, or published papers, and you may ask the instructors for help with Maple code. Other than that, you may not consult with anyone. Make 2 copies of your answers, one for each instructor.

1. (66 pts) The Leslie-Lefkowitch matrix for a population model with classes I, II, III, IV is

$$A = \begin{bmatrix} 0 & 0.5 & 4.0 & 0 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.3 & 0.2 \end{bmatrix}$$

a. Does this model describe age classes or stages? Why? (Suggestion: what is the significance of the last entry in the bottom row?) Which stages are reproductive?
b. If the current population consists of 100 individuals in class II, how many individuals will be in each class two time steps from now?
c. The dominant eigenvalue for this system is λ=1.53, and a corresponding eigenvector is [0.51, 0.3, 0.16, 0.04]. Over the long term does the population grow, decline, or remain stable? What is the long term age/stage distribution of the population in percent terms?

2. (66 pts) The plot below is $N_{t+1} = F(N_t)$



a) Based on the graph, what is the non-zero steady state N*?
b) If N_0=2, use the graph to compute N_4.
c) Discuss the stability properties of the two equilibria, and the local behavior of the system near the two equilibria in terms of the slope of the appropriate tangent line. Draw the tangent line and determine its slope using the axes and a ruler.

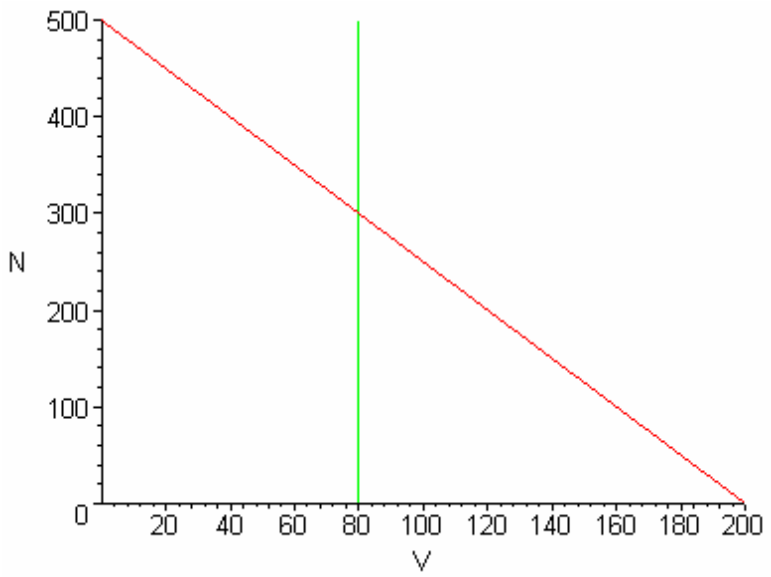
3. (66 pts) Consider a predator-victim system described by:

$$\frac{dV}{dt} = aV(1 - \frac{V}{K}) - bVN$$

$$\frac{dN}{dt} = cVN - dN$$

All parameters (a,b,c,d,K) are positive.

a. What happens to the victim when the predator population is zero? What happens to the predator when the victim population is zero? (do the populations increase, decrease, or remain constant)
b. The following is a phase plot of the system where a=0.5, b=0.001,c=0.005, d=0.4,K=200.



c. Which nullcline is dN/dt=0, and which is dV/dt=0?
d. If you start a population in each of the 4 regions, what happens to H (increase, decrease, remain constant)? What happens to V in the same starting conditions? Use arrows on the plot to indicate the direction of change.
e. What is the Jacobian matrix of this system in symbolic form?
f. What numerical values does the Jacobian have, when evaluated at the two-species equilibrium?
g. What do its eigenvalues tell you about the local stability of the system at the two-species equilibrium? Explain your reasoning.