

Prologue

BIOL 570, Principles of Ecology

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Aritmetic average

The aritmetic average is very familiar to us, for example,

$$\hat{r} = \frac{r_0 + r_1 + r_2 + \dots + r_n}{n}.$$

Example

Say that your paychecks varied randomly from one to the next. Your average pay is the amount such that a series of checks of that amount would give the same total pay as the random checks.

Check	Amount
1	\$601.00
2	\$742.00
3	\$661.00
4	\$541.00
Total:	\$2545.00
Average:	\$636.25

Table 1: Four random paychecks.

The series of checks, \$636.25, \$636.25,\$636.25, \$636.25, would give the same total pay as the random checks.

Interest rates and averages

The interest rate that your bank pays varies randomly. Say that they change it once each year and the rates over the last four years have been, 1.26, 1.31, 1.37, 1.42, (these are obviouly very generous interest rates).

Let's assume that your account had \$1,000.00 at the beginning of this period. How much money would you have now?

$$\begin{aligned}\text{money now} &= \$1000.00 \times 1.26 \times 1.31 \times 1.37 \times 1.42 \\ &= \$1000.00 \times 3.21108 \\ &= \$3211.08\end{aligned}$$

Now, what is the "average" interest rate such that you would earn the same amount of money after four years? A reasonable guess would be to use the arithmetic average.

$$\begin{aligned}\text{arithmetic average interest rate} &= \frac{1.26 + 1.31 + 1.37 + 1.42}{4} \\ &= 1.34\end{aligned}$$

Using this number for the "average" interest rate yields,

$$\begin{aligned}\text{money now} &= \$1000.00 \times 1.34 \times 1.34 \times 1.34 \times 1.34 \\ &= \$1000.00 \times 3.22418 \\ &= \$3224.18\end{aligned}$$

However, that answer is wrong, yielding too much money after four years.

The geometric average

The geometric average is what is needed to get the right answer.

$$\text{geometric average} = (r_0 \times r_1 \times r_2 \times \dots \times r_n)^{1/n}.$$

Returning to our example,

$$\begin{aligned}\text{geometric average} &= (1.26 \times 1.31 \times 1.37 \times 1.42)^{1/4} \\ &= 1.3386365.\end{aligned}$$

Next, this "average" interest rate is used to calculate the yield after four years,

$$\begin{aligned}\text{money now} &= \$1000.00 \times 1.3386365 \times 1.3386365 \times 1.3386365 \times 1.3386365 \\ &= \$1000.00 \times 3.21107724 \\ &= \$3211.08.\end{aligned}$$

Which is the correct answer.