

Metapopulations

BIOL 570, Principles of Ecology

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Random growth rates

Time	Patch 0 growth rate	Patch 1 growth rate
0	r_0^0	r_0^1
1	r_1^0	r_1^1
2	r_2^0	r_2^1
3	r_3^0	r_3^1
...
t	r_t^0	r_t^1

Table 1: Random population growth rates in two patches over time. The subscripts denote time and the superscripts denote the patch.

Important question

How does the total population size, T , respond to dispersal? Assume that each patch receives an equal portion of the offspring every generation. Assume that the initial population size in patch 0 is N_0 and in patch 1 is M_0 and that the total population size in the system is $T_0 = N_0 + M_0$. After dispersal there will be $T_0/2 = (N_0 + M_0)/2$ in each patch. After a round of reproduction the total population in the whole system at time, $t = 1$, is,

$$T_1 = r_0^0 \frac{N_0 + M_0}{2} + r_0^1 \frac{N_0 + M_0}{2}$$

Repeating this procedure again and again until time t and doing some algebra yields,

$$T_t = (N_0 + M_0) \frac{r_0^0 + r_0^1}{2} \frac{r_1^0 + r_1^1}{2} \frac{r_2^0 + r_2^1}{2} \times \dots \times \frac{r_t^0 + r_t^1}{2}. \quad (1)$$

This formula gives the total population size of the system at any time, t , for a two patch system that is subject to independent random growth rates in each patch.

Another important question

How does the total population size, T , respond to the absence of dispersal? The total in patch 0 at time, t , is given by,

$$N_0 r_0^0 r_1^0 r_2^0 r_3^0 \times \dots \times r_t^0,$$

and the total in patch 1 at time, t , is given by,

$$M_0 r_0^1 r_1^1 r_2^1 r_3^1 \times \dots \times r_t^1.$$

And the total in both patches at time, t , is simply the sum of patch totals,

$$N_0 r_0^0 r_1^0 r_2^0 r_3^0 \times \dots \times r_t^0 + M_0 r_0^1 r_1^1 r_2^1 r_3^1 \times \dots \times r_t^1. \quad (2)$$

A few random numbers

1.66843 1.8241 1.01645 1.62852 1.26402 1.09783
 0.973927 0.900406 0.874183 0.933341 0.896478 1.00968
 0.763558 0.986232 0.945299 1.11354 0.592372 1.29637
 0.56353 0.409669 0.774728 0.603114 0.871491 0.910697
 0.707878 0.876125 1.46631 0.377552 1.02957 1.11138
 1.44401 1.32955 1.13706 1.27855 1.09276 0.688682
 1.20921 1.41023 0.911083 1.11842 0.62815 1.40461
 1.25698 0.594608 0.870333 0.665985 1.52553 0.779189
 1.66372 0.970893 1.09644 1.08426 1.11706 1.05739
 1.09071 0.639512 0.954338 0.68897 1.14331 0.785132
 0.876729 1.3824 0.826847 0.963877 0.781185 0.815538
 0.989652 0.761915 0.579584 1.30105 0.919586 0.965255
 0.646477 1.12085 0.696719 1.41558 1.14149 0.746389
 0.903224 1.28939 1.26384 2.09985 0.5733 1.27088
 1.18769 1.13988 0.991027 1.09853 0.825524 1.29555
 1.12712 1.14021 1.93442 0.919768 1.09885 0.966649
 1.02262 1.01462 1.44461 0.699004 1.56569 0.666663
 1.17067 0.765885 0.744713 0.964647

These numbers are drawn from a gamma distribution with expectation 1.01 and standard deviation 0.3.